

The Feynman-Dyson Propagators for Neutral Particles (Local or Non-local?)

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Abstract An analog of the $S = 1/2$ Feynman-Dyson propagator is presented in the framework of the $S = 1$ Weinberg's theory. The basis for this construction is the concept of the Weinberg field as a system of four field functions differing by parity and by dual transformations. Next, we analyze the recent controversy in the definitions of the Feynman-Dyson propagator for the field operator containing the $S = 1/2$ self/anti-self charge conjugate states in the papers by D. Ahluwalia et al. and by W. Rodrigues Jr. et al. The solution of this mathematical controversy is obvious. It is related to the necessary doubling of the Fock Space (as in the Barut and Ziino works), thus extending the corresponding Clifford Algebra. However, the logical interrelations of different mathematical foundations with the physical interpretations are not so obvious (Physics should choose only one correct formalism: it is not clear, why two correct mathematical formalisms (which are based on the same postulates) lead to different physical results?)

1 The Weinberg Propagators.

Accordingly to the Feynman-Dyson-Stueckelberg ideas, a causal propagator has to be constructed by using the formula in Ref. [1, p.91] In the $S = 1/2$ Dirac theory it results to

$$S_F(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{\hat{k} + m}{k^2 - m^2 + i\epsilon}, \quad (1)$$

provided that the constant a and b are determined by imposing $(i\hat{\partial}_2 - m)S_F(x_2, x_1) = \delta^{(4)}(x_2 - x_1)$ in [1, p.91]. Namely, $a = -b = 1/i$.

However, attempts to construct the covariant propagator in this way have failed in the framework of the Weinberg theory, Ref. [2]. It is a generalization of the Dirac ideas to higher spins. For instance, on the page B1324 of Ref. [2] Weinberg writes:

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“Unfortunately, the propagator arising from Wick’s theorem is NOT equal to the covariant propagator except for $S = 0$ and $S = 1/2$. The trouble is that the derivatives act on the $\varepsilon(x) = \theta(x) - \theta(-x)$ in $\Delta^C(x)$ as well as on the functions¹ Δ and Δ_1 . This gives rise to extra terms proportional to equal-time δ functions and their derivatives. . . The cure is well known: . . . compute the vertex factors using only the original covariant part of the Hamiltonian \mathcal{H} ; do not use the Wick propagator for internal lines; instead use the covariant propagator. The propagator proposed in Ref. [3] is the causal propagator. However, the old problem persists: the Feynman-Dyson propagator is not the Green function of the Weinberg equation. As mentioned, the covariant propagator proposed by Weinberg propagates kinematically spurious solutions [3].

The aim of my paper is to consider the problem of constructing the propagator in the framework of the model given in [4]. The concept of the Weinberg field ‘doubles’ has been proposed there. It is based on the equivalence between the Weinberg field and the antisymmetric tensor field, which can be described by both $F_{\mu\nu}$ and its dual $\tilde{F}_{\mu\nu}$. These field operators may be used to form a parity doublet. An essential ingredient of my consideration is the idea of combining the Lorentz and the dual transformation. The set of four equations has been proposed in Ref. [4].

The simple calculations give

$$u_1^{(1)} \bar{u}_1^{(1)} = \frac{1}{2} \begin{pmatrix} m^2 & S_p \otimes S_p \\ \bar{S}_p \otimes \bar{S}_p & m^2 \end{pmatrix}, u_2^{(1)} \bar{u}_2^{(1)} = \frac{1}{2} \begin{pmatrix} -m^2 & S_p \otimes S_p \\ \bar{S}_p \otimes \bar{S}_p & -m^2 \end{pmatrix}, \quad (2)$$

$$u_1^{(2)} \bar{u}_1^{(2)} = \frac{1}{2} \begin{pmatrix} -m^2 & \bar{S}_p \otimes \bar{S}_p \\ S_p \otimes S_p & -m^2 \end{pmatrix}, u_2^{(2)} \bar{u}_2^{(2)} = \frac{1}{2} \begin{pmatrix} m^2 & \bar{S}_p \otimes \bar{S}_p \\ S_p \otimes S_p & m^2 \end{pmatrix}, \quad (3)$$

where

$$S_p = m + (\mathbf{S} \cdot \mathbf{p}) + \frac{(\mathbf{S} \cdot \mathbf{p})^2}{E + m}, \quad \bar{S}_p = m - (\mathbf{S} \cdot \mathbf{p}) + \frac{(\mathbf{S} \cdot \mathbf{p})^2}{E + m}. \quad (4)$$

And, u – are the 6-component objects for spin 1, which are solutions of the Weinberg “double” equations in the momentum space. One can conclude: the generalization of the notion of causal propagators is admitted by using the ‘Wick’s formula’ for the time-ordered particle operators provided that $a = b = 1/4im^2$. It is necessary to consider all four equations. Obviously, this is related to the 12-component formalism, which I presented in [4].

The $S = 1$ analogues of the formula (1) for the Weinberg propagators follow immediately. In the Euclidean metrics they are:

$$S_F^{(1)}(p) \sim -\frac{1}{i(2\pi)^4(p^2 + m^2 - i\varepsilon)} [\gamma_{\mu\nu} p_\mu p_\nu - m^2], \quad (5)$$

¹ In the cited paper $\Delta_1(x) \equiv i[\Delta_+(x) + \Delta_+(-x)]$ and $\Delta(x) \equiv \Delta_+(x) - \Delta_+(-x)$ have been used. $i\Delta_+(x) \equiv \frac{1}{(2\pi)^3} \int \frac{d^3p}{2E_p} \exp(ipx)$ is the particle Green function.

$$S_F^{(2)}(p) \sim -\frac{1}{i(2\pi)^4(p^2 + m^2 - i\varepsilon)} [\gamma_{\mu\nu} p_\mu p_\nu + m^2] , \quad (6)$$

$$S_F^{(3)}(p) \sim -\frac{1}{i(2\pi)^4(p^2 + m^2 - i\varepsilon)} [\tilde{\gamma}_{\mu\nu} p_\mu p_\nu + m^2] , \quad (7)$$

$$S_F^{(4)}(p) \sim -\frac{1}{i(2\pi)^4(p^2 + m^2 - i\varepsilon)} [\tilde{\gamma}_{\mu\nu} p_\mu p_\nu - m^2] . \quad (8)$$

$\gamma_{\mu\nu}$ are the covariantly defined 6×6 matrices of the $(1,0) \oplus (0,1)$ representation.

We should use the obtained set of Weinberg propagators (5,6,7,8) in the perturbation calculus of scattering amplitudes. In Ref. [6] the amplitude for the interaction of two $2(2S+1)$ bosons has been obtained on the basis of the use of one field only and it is obviously incomplete, see also Ref. [5]. But, it is interesting that the spin structure was proved there to be the same, regardless we consider the two-Dirac-fermion interaction or the two-Weinberg($S=1$)-boson interaction. However, the denominator slightly differs in the cited papers [6] from the fermion-fermion case. More accurate considerations of the fermion-boson and boson-boson interactions in the framework of the Weinberg theory has been reported elsewhere [7].

2 The Self/Anti-self Charge Conjugate Construct in the $(1/2, 0) \oplus (0, 1/2)$ Representation.

The first formulations with doubling solutions of the Dirac equations have been presented in Refs. [10], and [11]. The group-theoretical basis for such doubling has been given in the papers by Gelfand, Tsetlin and Sokolik [12], who first presented the theory in the 2-dimensional representation of the inversion group in 1956 (later called as ‘the Bargmann-Wightman-Wigner-type quantum field theory’ in 1993). M. Markov wrote long ago *two* Dirac equations with the opposite signs at the mass term [10]:

$$[i\gamma^\mu \partial_\mu - m] \Psi_1(x) = 0 , \quad (9)$$

$$[i\gamma^\mu \partial_\mu + m] \Psi_2(x) = 0 . \quad (10)$$

In fact, he studied all properties of this relativistic quantum model (while he did not know yet the quantum field theory in 1937). Next, he added and subtracted these equations. What did he obtain?

$$i\gamma^\mu \partial_\mu \varphi(x) - m\chi(x) = 0 , \quad i\gamma^\mu \partial_\mu \chi(x) - m\varphi(x) = 0 . \quad (11)$$

Thus, φ and χ solutions can be presented as some superpositions of the Dirac 4-spinors $u-$ and $v-$. These equations, of course, can be identified with the equations for the Majorana-like $\lambda-$ and $\rho-$ spinors, which we presented in Ref. [8, 9]. The equations can be written in the 8-component form as follows:

$$[i\Gamma^\mu \partial_\mu - m] \Psi_{(+)}(x) = 0, \quad [i\Gamma^\mu \partial_\mu + m] \Psi_{(-)}(x) = 0. \quad (12)$$

The signs at the mass terms depend on how do we choose the “positive”- and “negative”- energy solutions. For instance,

$$\Psi_{(+)}(x) = \begin{pmatrix} \rho^A(x) \\ \lambda^S(x) \end{pmatrix}, \Psi_{(-)}(x) = \begin{pmatrix} \rho^S(x) \\ \lambda^A(x) \end{pmatrix}, \Gamma^\mu = \begin{pmatrix} 0 & \gamma^\mu \\ \gamma^\mu & 0 \end{pmatrix}. \quad (13)$$

It is easy to find the corresponding projection operators, and the Feynman-Dyson-Stueckelberg propagator.

You may say that all this is just related to the spin-parity basis rotation (unitary transformations). However, in the previous papers I explained: the connection with the Dirac spinors has been found [9, 13], provided that the 4-spinors have the same physical dimension. Thus, we can see that the two 4-spinor systems are connected by the unitary transformations, and this represents itself the rotation of the spin-parity basis. However, it is usually assumed that the λ - and ρ - spinors describe the neutral particles, meanwhile u - and v - spinors describe the charged particles. Kirchbach [13] found the amplitudes for neutrinoless double beta decay ($00\nu\beta$) in this scheme. It is obvious from that connections that there are some additional terms comparing with the standard formulation.

Barut and Ziino [11] proposed yet another model. They considered γ^5 operator as the operator of the charge conjugation. The concept of the doubling of the Fock space has been developed in the Ziino works (cf. [12, 4]) in the framework of the quantum field theory. In their case the self/anti-self charge conjugate states are simultaneously the eigenstates of the chirality. Next, our formulation with the λ - and ρ - spinors naturally lead to the Ziino-Barut scheme of massive chiral fields.

3 The Controversy.

I cite Ahluwalia *et al.*, Ref. [14]: “To study the locality structure of the fields $\Lambda(x)$ and $\lambda(x)$, we observe that field momenta are

$$\Pi(x) = \frac{\partial \mathcal{L}^\Lambda}{\partial \dot{\Lambda}} = \frac{\partial}{\partial t} \bar{\Lambda}(x), \quad (14)$$

and similarly $\pi(x) = \frac{\partial}{\partial t} \bar{\lambda}(x)$. The calculational details for the two fields now differ significantly. We begin with the evaluation of the equal time anticommutator for $\Lambda(x)$ and its conjugate momentum

$$\{\Lambda(\mathbf{x}, t), \Pi(\mathbf{x}', t)\} = i \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2m} e^{i\mathbf{p} \cdot (\mathbf{x} - \mathbf{x}')}$$

$$\times \underbrace{\sum_{\alpha} \left[\xi_{\alpha}(\mathbf{p}) \bar{\xi}_{\alpha}(\mathbf{p}) - \zeta_{\alpha}(-\mathbf{p}) \bar{\zeta}_{\alpha}(-\mathbf{p}) \right]}_{=2m[I+\mathcal{G}(\mathbf{p})]}.$$

The term containing $\mathcal{G}(\mathbf{p})$ vanishes only when $\mathbf{x} - \mathbf{x}'$ lies along the z_e axis (see Eq. (24) [therein], and discussion of this integral in Ref. [15])

$$\mathbf{x} - \mathbf{x}' \text{ along } z_e : \quad \{\Lambda(\mathbf{x}, t), \Pi(\mathbf{x}', t)\} = i\delta^3(\mathbf{x} - \mathbf{x}')I \quad (15)$$

The anticommutators for the particle/antiparticle annihilation and creation operators suffice to yield the remaining locality conditions,

$$\{\Lambda(\mathbf{x}, t), \Lambda(\mathbf{x}', t)\} = O, \quad \{\Pi(\mathbf{x}, t), \Pi(\mathbf{x}', t)\} = O. \quad (16)$$

The set of anticommutators contained in Eqs. (15) and (16) establish that $\Lambda(x)$ becomes local along the z_e axis. For this reason we call z_e as the dark axis of locality.”

Next, I cite Rodrigues *et al.*, Ref. [16]: “We have shown through explicitly and detailed calculation that the integral of $\mathcal{G}(\mathbf{p})$ appearing in Eq.(42) of [14] is null for $\mathbf{x} - \mathbf{x}'$ lying in three orthonormal spatial directions in the rest frame of an arbitrary inertial frame $\mathbf{e}_0 = \partial/\partial t$.”

This shows that the existence of elko spinor fields does not implies in any break-down of locality concerning the anticommutator of $\{\Lambda(\mathbf{x}, t), \Pi(\mathbf{x}', t)\}$ and moreover does not implies in any preferred spacelike direction field in Minkowski spacetime.”

Who is right? In 2013 W. Rodrigues [17] changed a bit his opinion. He wrote: “When $\Delta_z \neq 0$, $\mathcal{G}(\mathbf{x} - \mathbf{x}')$ is null the anticommutator is local and thus there exists in the elko theory as constructed in [14] an infinity number of “locality directions”. On the other hand $\mathcal{G}(\mathbf{x} - \mathbf{x}')$ is a distribution with support in $\Delta_z = 0$. So, the directions $\Delta = (\Delta_x, \Delta_y, 0)$ are nonlocal in each arbitrary inertial reference frame \mathbf{e}_0 chosen to evaluate $\mathcal{G}(\mathbf{x} - \mathbf{x}')$ ”, thus accepting the Ahluwalia *et al.* viewpoint. See the cited papers for the notation.

Meanwhile, I suggest to use the 8-component formalism (see the Section 2) in the similarity with the 12-component formalism of the Section 1. If we calculate

$$\begin{aligned} S_F^{(+,-)}(x_2, x_1) &= \int \frac{d^3k}{(2\pi)^3} \frac{m}{E_k} \sum_{\sigma} \left[\theta(t_2 - t_1) a \Psi_+^{\sigma}(k) \otimes \bar{\Psi}_+^{\sigma}(k) e^{-ikx} + \right. \\ &\quad \left. + \theta(t_1 - t_2) b \Psi_-^{\sigma}(k) \otimes \bar{\Psi}_-^{\sigma}(k) e^{ikx} \right] = \\ &= \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{(\hat{k} + m) \otimes I_2}{k^2 - m^2 + i\epsilon}, \end{aligned} \quad (17)$$

we easily come to the result that the corresponding Feynman-Dyson propagators are local in the sense: $[i\Gamma_{\mu} \partial_2^{\mu} \mp m] S_F^{(+,-)}(x_2 - x_1) = \delta^{(4)}(x_2 - x_1)$. However, again: Physics should choose only one correct formalism. It is not clear, why two correct mathematical formalisms lead to different physical results?

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